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GEOMETRY.

135. Proposed by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

If a hyperbola be described touching the four sides of a quadrilateral which is inscribed in a circle, and one focus lie on the circle, the other focus will also lie on the circle.

Solution by the PROPOSER.

Using quadrilinear notation, the equation to the circle circumscribing the quadrilateral whose sides are given by $\alpha=0, \beta=0, \gamma=0, \delta=0$, is $\alpha\gamma=\beta\delta\dots(1)$.

Now, it is well known that if the coördinates of one focus of a conic tangent to a given line be $\alpha', \beta', \gamma', \delta'$, those of the other focus are proportional to $1/\alpha', 1/\beta', 1/\gamma', 1/\delta'$.

But by the problem, $\alpha', \beta', \gamma', \delta'$ is on (1); then $\alpha'\gamma'=\beta'\delta'\dots(2)$, or

$$\frac{1}{\alpha'\gamma'} = \frac{1}{\beta'\delta'} \dots (3).$$

Substituting the reciprocals in (1) gives (3) also, and proves the theorem.

136. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Professor of Mathematics, Central High School, Dallas, Tex.

Construct a triangle having given the base, the median line to the base, and the difference of the base angles.

I. Solution by B. L. REMICK, Instructor of Mathematics, Bradley Institute, Peoria, Ill.

Let $LM=a$ =the base, $CP=m$ =median to the base, $\alpha-\beta$ =difference of base angles.

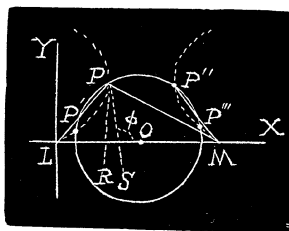
Then vertex P of required triangle lies on circle about C (mid point of LM) as center with radius m ; it also lies on the locus of point of intersection of straight lines through L, M forming angles with base having the required constant difference. We propose to show that this latter locus is an equilateral hyperbola and that our problem has therefore four solutions corresponding to the four common points of the circle and hyperbola.

$\angle RPS$ between the perpendicular and angle bisector $=\frac{1}{2}(\alpha-\beta)$ by a well known result in geometry; and hence we have to consider the locus of intersection of two straight lines passing through two given points L, M so that the angle bisector remains parallel to itself.

Let coördinates of P be (x_1, y_1) .

Equation PL is $y_1x - x_1y = 0$.

Equation PM is $y_1x + (a - x_1)y - ay_1 = 0$.



Equation *PS* is $\frac{y_1x-x_1y}{\sqrt{(y_1^2+x_1^2)}} = \frac{y_1x+(a-x_1)y-ay_1}{\sqrt{[y_1^2+(a-x_1)^2]}}$

Since $\tan \varphi$ has a constant value we obtain

$$\frac{\frac{y_1}{\sqrt{[y_1^2+(a-x_1)^2]}} - \frac{y_1}{\sqrt{(y_1^2+x_1^2)}}}{\frac{x_1}{\sqrt{(y_1^2+x_1^2)}} + \frac{a-x_1}{\sqrt{[y_1^2+(a-x_1)^2]}}} = k \text{ (constant).}$$

Simplifying, we have

$$(kx_1+y_1-ka)\sqrt{(y_1^2+x_1^2)} = (kx_1+y_1)\sqrt{[y_1^2+(a-x_1)^2]}.$$

Squaring and reducing, we obtain the form

$$2kx_1^2 + 2(1-k^2)x_1y_1 - ky_1^2 - 2akx_1 + (ak^2 - a)y_1 = 0,$$

which shows the locus a conic.

Note points *L*, *M*, viz., (0, 0), (*a*, 0) are on locus.

Testing the discriminant to determine the nature of the conic, we have

$$AB - H^2 = -4k^2 - (1-k^2)^2 = -(1+k^2)^2 < 0$$

Hence the curve is a hyperbola.

To examine the locus farther by proceeding to standard form of equation, make use of transformation $x=x'+x_0$, $y=y'+y_0$. equate to zero coefficients of x' , y' in the equation found and obtain as first reduced form

$$4kx'^2 + 4(1-k^2)x'y' - 4ky'^2 - a^2k = 0.$$

Again using formulæ $A'+B'=0$, $A'B' = -(1+k^2)^2$ obtain as final reduced equation

$$(1+k^2)x''^2 - (1+k^2)y''^2 = a^2k,$$

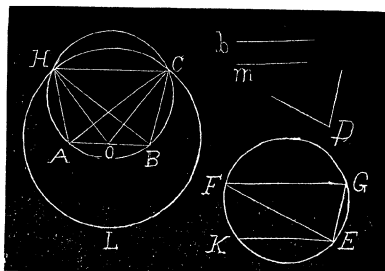
indicating an equilateral hyperbola.

Solved in a similar manner by *H. C. WHITAKER*.

II. Solution by *G. B. M. ZERR, A.M., Ph.D.*, The Temple College, Philadelphia, Pa.; *ALOIS K. KOVARIK*, Decorah Institute, Decorah, Ia.; and *J. SCHEFFER, A. M.*, Hagerstown, Md.

Let b =base, m =median, D =difference of the base angles.

In any circle *FGEG*, find a segment *FKEG* containing an angle *FEG* equal to $\angle D$ and draw *EK* parallel to *FG*.



Take $AB=b$; with O the center of AB as center and a radius equal to m , describe the circle HCL . Construct the angles ABH and HBC equal to the angles KEF and FEG , respectively.

Through $ABCH$ describe a circle. Join AH, OH, OC, AC . Then ABC or ABH is the required triangle, since $AB=b, OH=OC=m, \angle HCB=\angle HAC=D$.

CALCULUS.

102. Proposed by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A right cone has its vertex at the focus of a paraboloid of revolution, the axis of the cone perpendicular to the axis of the paraboloid. Find the volume common to both.

Solution by the PROPOSER.

Let $x^2+y^2=c^2z^2$, be the equation to the cone. $y^2+z^2=4a(a+x)$, be the equation to the paraboloid.

The limits of z are $z=\frac{\sqrt{(x^2+y^2)}}{c}$ to $z=\sqrt{(4a^2+4ax-y^2)}$;

of y , 0 and $\sqrt{\frac{4a^2c^2+4ac^2x-x^2}{1+c^2}}=y_1$;

of x , $2ac[c-\sqrt{(1+c^2)}]=x_2$ to $2ac[c+\sqrt{(1+c^2)}]=x_1$.

$$\begin{aligned} \therefore V &= 2 \int_{x_2}^{x_1} \left(\sqrt{(4a^2+4ax-y^2)} - \frac{\sqrt{(x^2+y^2)}}{c} \right) dx dy \\ &= \int_{x_2}^{x_1} \left[\left[y \sqrt{(4a^2+4ax-y^2)} + 4a(a+x) \sin^{-1} \left(\frac{y}{2\sqrt{(a(a+x))}} \right) - \frac{y \sqrt{(x^2+y^2)}}{c} \right] \right. \\ &\quad \left. - \frac{x^2}{c} \log[y + \sqrt{(x^2+y^2)}] \right]_{y_1}^{y_2} dx = 4a \int_{x_2}^{x_1} (a+x) \sin^{-1} \left(\frac{\sqrt{(4ac^2(a+x)-x^2)}}{2\sqrt{(a(1+c^2)(a+x))}} \right) dx \\ &\quad - \frac{1}{c} \int_{x_2}^{x_1} x^2 \log \left(\frac{\sqrt{(4ac^2(a+x)-x^2)} + c(2a+x)}{x\sqrt{(1+c^2)}} \right) dx. \\ \therefore V &= \left[2a(a+x)^2 \sin^{-1} \left(\frac{\sqrt{(4ac^2(a+x)-x^2)}}{2\sqrt{(a(1+c^2)(a+x))}} \right) \right. \\ &\quad \left. - \frac{x^3}{3c} \log \left(\frac{\sqrt{(4ac^2(a+x)-x^2)} + c(2a+x)}{x\sqrt{(1+c^2)}} \right) \right]_{x_2}^{x_1} + a \int_{x_2}^{x_1} \frac{x(a+x) dx}{x \sqrt{(4ac^2(a+x)-x^2)}} \\ &\quad + \frac{1}{3} \int_{x_2}^{x_1} \frac{x^3 dx}{\sqrt{(4ac^2(a+x)-x^2)}} - \frac{1}{3c} \int_{x_2}^{x_1} x^2 dx \end{aligned}$$